$$\div\,\pi\Big[\tfrac{1}{3}(1-x)^3\sin^{-1}\frac{x}{1-x}-\frac{\pi}{6}(1-x)^3$$

$$+\tfrac{1}{12}(1-2x)^{\frac{1}{2}}+\tfrac{2}{9}(1-2x)^{\frac{3}{2}}-\tfrac{1}{12}(1-2x)^{\frac{5}{2}}\Big]_0^{\frac{1}{2}}$$

$$=\frac{4}{3\pi-4}\left(1-\frac{1}{\pi}-\frac{2}{\pi}\log 2\right).$$

MISCELLANEOUS.

FURTHER REMARK ON PROBLEM 90.

The results of the problem may be put in a better form as follows : e must be a function of s, say $e=f(s)$. For a *continuous* x,

$$f(s+x)=f(s)+x\Delta f(s)+\frac{x(x-1)}{2\,!}\Delta^2 f(s)+\frac{x(x-1)(x-2)}{3\,!}\Delta^3 f(s).$$

Put $s=0$, and substitute for $f(0)$, $\Delta f(0)$, $\Delta^2 f(0)$, $\Delta^3 f(0)$, their values 21, $\frac{7}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, respectively, and

$$f(x)=21+\tfrac{41}{12}x+\frac{x^3}{12};$$

replacing x by a *continuous* s,

$$e=f(s)=21+\frac{41s}{12}+\frac{s^3}{12},$$

which determines once for all the functional relation between any value of s and e. This result is somewhat analagous to the primitive functional relation found from a differential equation, only here difference coefficients enter instead of differential coefficients, and the work is infinitely simpler. Of course the same results could have been obtained by La Grange's formula of interpolation. In order to use the above formula for calculating e for any distance s, 100 yards must be taken as the unit.

E. D. ROE, JR.

93. **Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.**

Prove that $-(\sqrt{-1})^{\sqrt{-1}}=e^{(\sqrt{-1}-\frac{1}{2})\pi}$.

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

In the Napierian system of logarithms we always have $a^n=e^{n\log a}$....(1).

Put $\sqrt{-1}=i$; then $-1=i^2$, and $-i^i=i^2.i^i=i^{i+2}$....(2).

In (1), putting $a=i$, $n=i+2$, gives $i^{i+2}=e^{(i+2)\log i}$ ....(3).

But $\log i=(2n\pi+\frac{1}{2}\pi)i$....(4); then (3) is, with $n=0$,

$$i^{i+2}=e^{(i+2)(\frac{1}{2}\pi)i}=e^{(i-\frac{1}{2})\pi}.$$

II. Solution by J. W. YOUNG, Oliver Graduate Scholar in Mathematics, Cornell University, Ithaca, N. Y.; GEORGE LILLEY, Ph. D., LL. D., Professor of Mathematics, State University, Eugene, Ore.; and HARRY S. VANDIVER, Bala, Pa.

Since $\cos\theta+i\sin\theta=e^{i\theta}$ $[i=\sqrt{-1}]$ we have $i=e^{i\frac{1}{2}\pi}$, $i^i=e^{-\frac{1}{2}\pi}$, $-i^i=e^{i\pi}.e^{-\frac{1}{2}\pi}$.

$$\text{or } -(\sqrt{-1})^{\sqrt{-1}}=e^{(\sqrt{-1}-\frac{1}{2})\pi}.$$

III. Solution by CHARLES PURYEAR, Department of Mathematics, Agricultural and Mechanical College, College Station, Texas.

$$e^x=1+x+\frac{x^2}{2\,!}+\frac{x^3}{3\,!}+\frac{x^4}{4\,!}+\ldots.(1).$$

$$e^{-x}=1-x+\frac{x^2}{2\,!}-\frac{x^3}{3\,!}+\frac{x^4}{4\,!}-\ldots.(2).$$

$$\therefore e^x+e^{-x}=2(1+\frac{x^2}{2\,!}+\frac{x^4}{4\,!}+\ldots.)\ldots.(3).$$

Replacing x by ix where $i=\sqrt{-1}$,

$$e^{ix}+e^{-ix}=2(1-\frac{x^2}{2\,!}+\frac{x^4}{4\,!}+\ldots.)\ldots.(4), \text{ or } e^{ix}+e^{-ix}=2\cos x\ldots.(5).$$

Let $x=\pi$, then $e^{\pi i}+e^{-\pi i}=-2$....(6).

Solving, $e^{\pi i}=-1$....(7).

Extracting the square root of each member of (7), $e^{(\frac{1}{2}\pi)i}=i$....(8).

Raising each member of (8) to the power of i, $e^{-\frac{1}{2}\pi}=(i)^i$ ...(9).

Multiplying (7) and (9), $e^{(i-\frac{1}{2})\pi}=-(i)^i$.

Also solved by *J. SCHEFFER*, *H. C. WHITAKER*, and *G. B. M. ZERR*.

94. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

The wall of a house, if its plane were extended, would cut the horison at an angle$=\beta°$ south of the true east point. The latitude of the place being$=\phi$, and the declination of the sun$=\delta$. When will the sun cease to shine through a window in that wall?

Solution by H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.

When the sun does not shine in the window its azimuth is $270°+\beta$. Let P be the pole, Z the zenith, and S the sun; then $PZ=co-\phi$, $PS=co-\delta$, angle $Z=90°+\beta$.

Therefore, by spherical trigonometry, $\sec\phi\tan\beta\sin P-\tan\phi\cos P=-\delta$.